

The investigation of detectability of the relic gravitational waves based on the WMAP-9 and Planck

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Abstract

The generated relic gravitational waves were underwent several stages of evolution of the universe such as inflation and reheating. These stages were affected on the shape of spectrum of the waves. As well known, at the end of inflation, the scalar field ϕ oscillates quickly around some point where potential $V(\phi) = \lambda\phi^n$ has a minimum. The end of inflation stage played a crucial role on the further evolution stages of the universe because particles were created and collisions of the created particles were responsible for reheating the universe. There is a general range for the frequency of the spectrum $\sim (0.3 \times 10^{-18} - 0.6 \times 10^{10})\text{Hz}$. It is shown that the reheating temperature can be affect on the frequency of the spectrum as well. There is constraint on the temperature from cosmological observations based on WMAP-9 and Planck. Therefore it is interesting to estimate allowed value of frequencies of the spectrum based on general range of reheating temperature like $\text{few MeV} \lesssim T_{rh} \lesssim 10^{16} \text{ GeV}$, WMAP-9 and Planck data then compare the spectrum with sensitivity of future detectors such as LISA, BBO and ultimate-DECIGO. The obtained results of this comparison give us some more chance for detection of the relic gravitational waves.

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I. INTRODUCTION

The relic gravitational waves generated in the early universe are very importance because they provide useful information about physics of the early universe. The waves were underwent several stages of evolution of the universe such as: inflation, reheating, radiation, matter and acceleration. The evolution of the universe at various epoch is affected the shape of the spectrum of the waves. Therefore it is unavoidable the consideration of different stages of evolution of the universe on the study of spectrum of the waves that are to be observed today. It is believed that the universe underwent a quasi exponential expansion in its early stages of evolution known as inflation stage. This stage is very important on the evolution of the universe because it provided the mechanism to formation of large scale structures in the universe. At the end of inflation, the scalar field ϕ oscillates quickly around some point where potential $V(\phi) = \lambda\phi^n$ has a minimum. The end of this stage played a crucial role on the further evolution stages of the universe because particles were created and collisions of the created particles were responsible for reheating the universe. The reheating was essential for the nucleosynthesis process since the inflation brought temperature of the universe below for the requirement of thermo nuclear reactions. Towards the end of inflation, during the reheating, the equation of state of energy for the universe is quite complicated and also model-dependent [1]. Hence a new stage that called z -stage is introduced to allow a general behaviour of reheating epoch [2].

There is a general range for the frequency of the spectrum $\sim(0.3 \times 10^{-18} - 0.6 \times 10^{10})$ Hz. It is shown that the reheating temperature (T_{rh}) can be affect on the frequency of the spectrum [3]. There is constraint on the T_{rh} from cosmological observations based on WMAP-9 and Planck. However, the reheating temperature must be larger than a few MeV [4], for the creation light elements, but less than the energy scale at the end of inflation, that is $T_{rh} \lesssim 10^{16}$ GeV. Therefore it is interesting to estimate allowed value of frequencies of the spectrum based on general range of T_{rh} , WMAP-9 and Planck data then compare the spectrum with sensitivity of future detectors such as LISA [5], Big Bang Observer (BBO) [6] and ultimate DECI-hertz Interferometer Gravitational-wave Observatory (ultimate -DECIGO) [7]. Hence the main purpose of this work is investigation of this comparison. The obtained results of this comparison give us some more chance for detection of the waves. We use the units $c = \hbar = k_B = 1$.

II. GRAVITATIONAL WAVES SPECTRUM IN EXPANDING UNIVERSE

The perturbed metric for a homogeneous isotropic flat Friedmann-Robertson-Walker universe can be written as

$$ds^2 = S^2(\eta)(d\eta^2 - (\delta_{ij} + h_{ij})dx^i dx^j), \quad (1)$$

where $S(\eta)$ is the cosmological scale factor, η is the conformal time and δ_{ij} is the Kronecker delta symbol. The h_{ij} are metric perturbations field contain only the pure gravitational waves and are transverse-traceless i.e; $\nabla_i h^{ij} = 0, \delta^{ij} h_{ij} = 0$.

The present study consider the shape of the spectrum of relic gravitational waves that generated by the expanding space time background. Thus the perturbed matter source is not taken into account. The gravitational waves are described with the linearized field equation given by

$$\nabla_\mu (\sqrt{-g} \nabla^\mu h_{ij}(\mathbf{x}, \eta)) = 0. \quad (2)$$

The tensor perturbations have two independent physical degrees of freedom like h^+ and h^\times and called polarization modes. To compute the spectrum of gravitational waves $h(\mathbf{x}, \eta)$, we express h^+ and h^\times in terms of the creation (a^\dagger) and annihilation (a) operators,

$$h_{ij}(\mathbf{x}, \eta) = \frac{\sqrt{16\pi} l_{pl}}{S(\eta)} \sum_{\mathbf{p}} \int \frac{d^3 k}{(2\pi)^{3/2}} \epsilon_{ij}^{\mathbf{p}}(\mathbf{k}) \times \frac{1}{\sqrt{2k}} [a_{\mathbf{k}}^{\mathbf{p}} h_{\mathbf{k}}^{\mathbf{p}}(\eta) e^{i\mathbf{k} \cdot \mathbf{x}} + a_{\mathbf{k}}^{\dagger \mathbf{p}} h_{\mathbf{k}}^{*\mathbf{p}}(\eta) e^{-i\mathbf{k} \cdot \mathbf{x}}], \quad (3)$$

where \mathbf{k} is the comoving wave number, $k = |\mathbf{k}|$, $l_{pl} = \sqrt{G}$ is the Planck's length and $\mathbf{p} = +, \times$ are polarization modes. The polarization tensor $\epsilon_{ij}^{\mathbf{p}}(\mathbf{k})$ is symmetric and transverse-traceless $k^i \epsilon_{ij}^{\mathbf{p}}(\mathbf{k}) = 0, \delta^{ij} \epsilon_{ij}^{\mathbf{p}}(\mathbf{k}) = 0$ and satisfy the conditions $\epsilon^{ij\mathbf{p}}(\mathbf{k}) \epsilon_{ij}^{\mathbf{p}'}(\mathbf{k}) = 2\delta_{\mathbf{p}\mathbf{p}'}$ and $\epsilon_{ij}^{\mathbf{p}}(-\mathbf{k}) = \epsilon_{ij}^{\mathbf{p}}(\mathbf{k})$. The a and a^\dagger satisfy $[a_{\mathbf{k}}^{\mathbf{p}}, a_{\mathbf{k}'}^{\dagger \mathbf{p}'}] = \delta_{\mathbf{p}\mathbf{p}'} \delta^3(\mathbf{k} - \mathbf{k}')$ and the initial vacuum state is defined as $a_{\mathbf{k}}^{\mathbf{p}}|0\rangle = 0$ for each \mathbf{k} and \mathbf{p} .

For a fixed wave number \mathbf{k} and a fixed polarization state \mathbf{p} the linearized wave eq.(2) gives

$$h_k'' + 2 \frac{S'}{S} h_k' + k^2 h_k = 0, \quad (4)$$

where prime means derivative with respect to the conformal time. Because the polarization states are same, we consider $h_k(\eta)$ without the polarization index.

Next, we rescale the field $h_k(\eta)$ by taking $h_k(\eta) = f_k(\eta)/S(\eta)$, where the mode functions $f_k(\eta)$ obey the minimally coupled Klein-Gordon equation

$$f_k'' + \left(k^2 - \frac{S''}{S}\right) f_k = 0. \quad (5)$$

The general solution of the above equation is a linear combination of the Hankel function with a generic power law for the scale factor $S = \eta^q$ given by

$$f_k(\eta) = A_k \sqrt{k\eta} H_{q-\frac{1}{2}}^{(1)}(k\eta) + B_k \sqrt{k\eta} H_{q-\frac{1}{2}}^{(2)}(k\eta). \quad (6)$$

For a given model of the universe, consisting of a sequence of successive scale factors with different q , we can obtain an exact solution $f_k(\eta)$ by matching its value and derivative at the joining points.

The approximate computation of the spectrum is calculated in two cases depending up on the waves that are outside or within of the barrier. For the gravitational waves outside barrier ($k^2 \gg S''/S$) the corresponding amplitude decrease as $h_k \propto 1/S(\eta)$ and for the waves inside the barrier ($k^2 \ll S''/S$), $h_k = C_k$ simply a constant [8].

The history of expansion of the universe can be obtained as follows: The inflation stage

$$S(\eta) = l_0 |\eta|^{1+\beta}, \quad -\infty < \eta \leq \eta_1, \quad (7)$$

where $1 + \beta < 0$, $\eta < 0$ and l_0 is a constant.

To make our analysis more general, we consider that the inflation stage was followed by some interval of the z-stage (z from Zeldovich). In fact this stage is quite general that considered by Zeldovich [9]. It can be governed by a softer than radiation matter, as well as by a stiffer than radiation matter [11]. Towards the end of inflation, during the reheating, the equation of state of energy in the universe can be quite complicated [1]. Hence this z-stage is introduced to allow a general reheating stage. Therefore we define the reheating stage in general form

$$S(\eta) = S_z (\eta - \eta_p)^{1+\beta_s}, \quad \eta_1 < \eta \leq \eta_s, \quad (8)$$

where $1 + \beta_s > 0$ [8].

The radiation-dominated stage

$$S(\eta) = S_e (\eta - \eta_e), \quad \eta_s \leq \eta \leq \eta_2, \quad (9)$$

and the matter-dominated stage

$$S(\eta) = S_m (\eta - \eta_m)^2, \quad \eta_2 \leq \eta \leq \eta_E, \quad (10)$$

where η_E is the time when the dark energy density ρ_Λ is equal to the matter energy density ρ_m .

The value of redshift z_E at η_E is $(1 + z_E) = S(\eta_0)/S(\eta_E) \sim 1.3$ from Planck collaboration [12], where η_0 is the present time.

The accelerating stage (up to the present)

$$S(\eta) = \ell_0 |\eta - \eta_a|^{-\gamma}, \quad \eta_E \leq \eta \leq \eta_0, \quad (11)$$

where γ is Ω_Λ dependent parameter, and Ω_Λ is the energy density contrast. We take $\gamma \simeq 1.97$ [13] for $\Omega_\Lambda = 0.73$ [14].

Except for β_s , there are ten constants in the expressions of $S(\eta)$. By the continuity conditions of $S(\eta)$ and $S'(\eta)$ at four given joining points η_1, η_s, η_2 , and η_E , one can fix only eight constants. The remaining two constants can be fixed by the normalization of S and the observed Hubble constant. We put $|\eta_0 - \eta_a| = 1$ for the normalization of S , which fixes the η_a , and the constant ℓ_0 is fixed by the following calculation,

$$\frac{\gamma}{H} \equiv \left(\frac{S^2}{S'} \right)_{\eta_0} = \ell_0, \quad (12)$$

where ℓ_0 is the Hubble radius at present and $H = 100 h \text{ km s}^{-1} \text{ Mpc}^{-1}$ with $h \simeq 0.704$ [14].

The physical wavelength is related to the comoving wave number as $\lambda \equiv 2\pi S(\eta)/k$. Assuming that the wave mode crosses the horizon of the universe when $\lambda/2\pi = 1/H$ [15], then wave number k_0 corresponding to the present Hubble radius is $k_0 = S(\eta_0)/\ell_0 = \gamma$. Also there is another wave number $k_E = \frac{S(\eta_E)}{1/H} = \frac{k_0}{1+z_E}$, whose corresponding wavelength at the time η_E is the Hubble radius $1/H$.

By matching S and S'/S at the joint points, one gets

$$l_0 = \ell_0 b \zeta_E^{-(2+\beta)} \zeta_2^{\frac{\beta-1}{2}} \zeta_s^\beta \zeta_1^{\frac{\beta-\beta_s}{1-\beta_s}}, \quad (13)$$

where $b \equiv |1 + \beta|^{-(2+\beta)}$, $\zeta_E \equiv \frac{S(\eta_0)}{S(\eta_E)}$, $\zeta_2 \equiv \frac{S(\eta_E)}{S(\eta_2)}$, $\zeta_s \equiv \frac{S(\eta_2)}{S(\eta_s)}$, and $\zeta_1 \equiv \frac{S(\eta_s)}{S(\eta_1)}$.

The power spectrum of gravitational waves is defined as

$$\int_0^\infty h^2(k, \eta) \frac{dk}{k} = \langle 0 | h^{ij}(\mathbf{x}, \eta) h_{ij}(\mathbf{x}, \eta) | 0 \rangle. \quad (14)$$

Substituting eq.(3) in eq.(14) and taking the contribution from each polarization is same, we get

$$h(k, \eta) = \frac{4l_{pl}}{\sqrt{\pi}} k |h(\eta)|. \quad (15)$$

Thus once the mode function $h(\eta)$ is known, the spectrum $h(k, \eta)$ follows.

The spectrum at the present time $h(k, \eta_0)$ can be obtained, provided the initial spectrum is specified. The initial condition is taken to be during the inflation. The wave with wave number k crossed over the horizon at a time η_i , when the wavelength $\lambda_i/2\pi = 1/H(\eta_i) = S(\eta_i)/k$ [15]. Now we choose the initial condition of the mode function h_k as $|h_k(\eta_i)| = 1/S(\eta_i)$. The initial amplitude of the power spectrum is

$$h(k, \eta_i) = 8\sqrt{\pi} \frac{l_{pl}}{\lambda_i}. \quad (16)$$

With $\lambda_i/2\pi = 1/H(\eta_i)$ it becomes

$$\frac{S'(\eta_i)}{S(\eta_i)} = k. \quad (17)$$

Therefore initial amplitude of the spectrum is given by

$$h(k, \eta_i) = A \left(\frac{k}{k_0} \right)^{2+\beta}, \quad (18)$$

where the constant A in eq.(18) can be determined by quantum normalization [8]:

$$A = 8\sqrt{\pi} \frac{l_{pl}}{l_0}. \quad (19)$$

Thus the amplitude of the spectrum for different ranges are given as follows [8], [16], [17], [18].

$$h(k, \eta_0) = A \left(\frac{k}{k_0} \right)^{2+\beta}, \quad k \leq k_E, \quad (20)$$

$$h(k, \eta_0) = A \left(\frac{k}{k_0} \right)^{\beta-\gamma} (1+z_E)^{\frac{-2-\gamma}{\gamma}}, \quad k_E \leq k \leq k_0, \quad (21)$$

$$h(k, \eta_0) = A \left(\frac{k}{k_0} \right)^{\beta} (1+z_E)^{\frac{-2-\gamma}{\gamma}}, \quad k_0 \leq k \leq k_2, \quad (22)$$

$$h(k, \eta_0) = A \left(\frac{k}{k_0} \right)^{1+\beta} \left(\frac{k_0}{k_2} \right) (1+z_E)^{\frac{-2-\gamma}{\gamma}}, \quad k_2 \leq k \leq k_s, \quad (23)$$

$$h(k, \eta_0) = A \left(\frac{k}{k_0} \right)^{1+\beta-\beta_s} \left(\frac{k_s}{k_0} \right)^{\beta_s} \left(\frac{k_0}{k_2} \right) (1+z_E)^{\frac{-2-\gamma}{\gamma}}, \quad k_s \leq k \leq k_1. \quad (24)$$

The factor A in all the spectra is determined with the CMB data of WMAP-9 [19]. The observed CMB anisotropies at lower multipoles is $\Delta T/T \simeq 0.44 \times 10^{-5}$ at $l \sim 2$ which corresponds to the largest scale anisotropies that have observed so far. Thus one can gets

$$h(k_0, \eta_0) = A(1+z_E)^{\frac{-2-\gamma}{\gamma}} \simeq 0.44 \times 10^{-5} r. \quad (25)$$

where r is tensor to scalar ratio [20] (see the appendix.(A) for more details). The parameter r taken ~ 0.1 from Planck collaboration [12]. However, there is a point in the interpretation of $\delta T/T$ at low multipoles. At present, the Hubble radius and Hubble diameter are ℓ_0 , $2\ell_0$ respectively. The corresponding physical wave length of k_E at present is $S(\eta_0)/k_E = \ell_0(1+z_E) \simeq 1.32\ell_0$, which is within $2\ell_0$ and is theoretically observable. So, instead of eq.(25), if $\delta T/T \simeq 0.44 \times 10^{-5}$ at $l = 2$ were taken as the amplitude of the spectrum at k_E , then we have $h(k_E, \eta_0) = 0.44 \times 10^{-5} \times r^{1/2}$ yielding a smaller A than that in eq.(25) [21, 22]. Also there is another normalization method that leads to decaying factor $S(\eta_i)/S(\eta_0)$ [23].

By taking ν as frequency, we can obtain $\nu_E = 0.30 \times 10^{-18}$ Hz, $\nu_0 = 0.36 \times 10^{-18}$ Hz, $\nu_2 = 1.48 \times 10^{-17}$ Hz, $\nu_s = 0.15 \times 10^8$ Hz.

The spectral energy density parameter $\Omega_g(\nu)$ of gravitational waves is defined through the relation $\rho_g/\rho_c = \int \Omega_g(\nu) \frac{d\nu}{\nu}$, where ρ_g is the energy density of the gravitational waves and ρ_c is the critical energy density. Therefore we have [8]

$$\Omega_g(\nu) = \frac{\pi^2}{3} h^2(k, \eta_0) \left(\frac{\nu}{\nu_0} \right)^2. \quad (26)$$

We assume that the space time is spatially flat $K = 0$ with $\Omega = 1$, then the fraction density of relic gravitational waves must be less than unity, $\rho_g/\rho_c < 1$. In order to ρ_g/ρ_c dose not exceed the level of 10^{-5} , the $\Omega_g(\nu_1) \simeq 10^{-6}$ in eq.(26) therefore we get $\nu_1 \simeq 3 \times 10^{10}$ Hz [8]. When the acceleration epoch is considered, the constraint becomes $\nu_1 \simeq 4 \times 10^{10}$ Hz [3].

So far we did not take the effect of reheating temperature on the spectrum of gravitational waves. Then we will consider this effect in the next section.

III. THE EFFECT OF REHEATING TEMPERATURE ON THE SPECTRUM

At the end of inflation, the scalar field ϕ oscillates quickly around some point where potential $V(\phi)$ has a minimum. It is found that the scalar field oscillations behave like a fluid with $p = \omega\rho$, where the average equation of state ω depends on the form of the potential $V(\phi)$ [24]. For $V(\phi) = \lambda\phi^n$, one has

$$\omega = \frac{n-2}{n+2}. \quad (27)$$

There is theoretical consideration during inflation and reheating stages for the equation of state $-1/3 < \omega < 1$ [3]. Due to eq.(27) the condition leads to $n > 1$. The upper bound

based on CMB observation for n gives $n < 2.1$ [25]. Therefore, we can write the range of n as follows

$$1 < n < 2.1. \quad (28)$$

There are two relations that connect the β and β_s with n [3]:

$$\beta_s = \frac{4-n}{2(n-1)}, \quad (29)$$

and

$$\beta = -2 - \frac{n}{2(n+2)}(1-n_s), \quad (30)$$

where n_s is scalar spectral index. And also we can write the T_{rh} as follows [26]

$$T_{rh} = 3.36 \times 10^{-68} \sqrt{\frac{1-n_s}{A_s}} \exp\left(\frac{6}{1-n_s}\right), \quad (31)$$

where A_s is amplitude of the scalar perturbations. For taking in account the effect of the T_{rh} on the spectrum, we can consider the following relations [3, 8]:

$$\zeta_s = \left(\frac{\nu_s}{\nu_2}\right) = \frac{S(\eta_2)}{S_{rec}} \frac{S_{rec}}{S(\eta_s)} = \frac{T_{rh}}{T_{CMB}(1+z_{eq})} \left(\frac{g_1}{g_2}\right)^{1/3}, \quad (32)$$

$$\zeta_1 = \left(\frac{\nu_1}{\nu_s}\right)^{(1+\beta_s)} = \frac{S(\eta_s)}{S(\eta_1)} = \frac{m_{pl}}{k_0^p} \left[\pi A_s (1-n_s) \frac{n}{2(n+2)} \right]^{1/2} \frac{T_{CMB}}{T_{rh}} \left(\frac{g_2}{g_1}\right)^{1/3} \exp\left[-\frac{n+2}{2(1-n_s)}\right], \quad (33)$$

where S_{rec} and T_{rec} stand for the scale factor and the temperature at the recombination, respectively and $k_0^p = 0.002 \text{ Mpc}^{-1}$ is pivot wavenumber. The $g_1 = 200$ and $g_2 = 3.91$ count the effective number of relativistic species contributing to the entropy during the reheating and that during recombination respectively [3]. Also we used $T_{rec} = T_{CMB}(1+z_{rec})$ with $T_{CMB} = 2.725 \text{ K} = 2.348 \times 10^{-13} \text{ GeV}$ [14].

Using eq.(31), the obtained T_{rh} for given n_s and A_s based on different types of the object for WMAP-9 (WMAP-9 +eCMB, WMAP-9 +eCMB+BAO, ...) and Planck (Planck+lensing, Planck+WMAP-9, ...) are shown in tables.(III,III) respectively. Therefore the acceptable range for T_{rh} corresponds to the range $\text{few MeV} \lesssim T_{rh} \lesssim 10^{16}$ are obtained as follows

$$0.0108 \leq T_{rh} \leq 7.8 \times 10^8 \text{ GeV}, \quad \text{WMAP} - 9, \quad (34)$$

The obtained T_{rh} is based on WMAP-9. The n_s , A_s and m are taken from [27] and W stands for

WMAP.				
<i>Object</i>	n_s	$A_s \times 10^9$	$m \times 10^{16}(GeV)$	$T_{rh}(GeV)$
W	0.972 ± 0.013	2.41 ± 0.10	$1.05 m_{pl}$	1.324×10^{29}
$W + eCMB$	0.9642 ± 0.0098	2.43 ± 0.084	$1.35 m_{pl}$	7.8×10^8
$W + eCMB + BAO$	$0.9579^{+0.0081}_{-0.0082}$	$2.484^{+0.073}_{-0.072}$	$1.61 m_{pl}$	0.0108
$W + eCMB + H_0$	$0.9690^{+0.0091}_{-0.0090}$	$2.396^{+0.079}_{-0.078}$	$1.16 m_{pl}$	1.378×10^{20}
$W + eCMB + BAO + H_0$	0.9608 ± 0.008	2.464 ± 0.072	$1.49 m_{pl}$	399.171

$$0.090 \leq T_{rh} \leq 3.4 \times 10^7 \quad GeV, \quad Planck. \quad (35)$$

Then we find the frequencies ν_s and ν_1 as function of the T_{rh} with help of eqs.(32–35). The obtained frequencies are smaller than their initial amount ($\nu_s = 0.15 \times 10^8$ Hz, $\nu_1 = 4 \times 10^{10}$ Hz) due to T_{rh} as follows

$$\begin{aligned} 0.78 \times 10^{-9} &\leq \nu_s \leq 0.55 \times 10^2 \quad Hz, \quad with \quad n \sim 1, \\ 0.47 \times 10^{-7} &\leq \nu_1 \leq 0.44 \times 10^3 \quad Hz, \quad with \quad n \sim 1, \end{aligned} \quad (36)$$

$$\begin{aligned} 0.78 \times 10^{-9} &\leq \nu_s \leq 0.55 \times 10^2 \quad Hz, \quad with \quad n \sim 2.1, \\ 0.28 \times 10^3 &\leq \nu_1 \leq 1.16 \times 10^6 \quad Hz, \quad with \quad n \sim 2.1, \end{aligned} \quad (37)$$

for WMAP-9 and

$$\begin{aligned} 0.65 \times 10^{-8} &\leq \nu_s \leq 2.45 \times 10^0 \quad Hz, \quad with \quad n \sim 1, \\ 0.33 \times 10^{-6} &\leq \nu_1 \leq 0.23 \times 10^2 \quad Hz, \quad with \quad n \sim 1, \end{aligned} \quad (38)$$

The obtained T_{rh} is based on Planck. The n_s , A_s and m are taken from [12]. The P , le , W , and hl stand for Planck, lensing, WMAP and HighL respectively.

<i>Object</i>	n_s	$\ln(10^{10} A_s)$	$m \times 10^{16} (GeV)$	$T_{rh} (GeV)$
P	0.9616 ± 0.0094	3.103 ± 0.072	$1.39 m_{pl}$	1.007×10^4
$P + le$	0.9635 ± 0.0094	3.085 ± 0.057	$1.30 m_{pl}$	3.4×10^7
$P + W$	0.9603 ± 0.0073	$3.089^{+0.024}_{-0.027}$	$1.42 m_{pl}$	61.867
$P + W + hl$	0.9585 ± 0.007	3.090 ± 0.025	$1.49 m_{pl}$	0.090
$P + le + W + hl$	0.9641 ± 0.0063	3.087 ± 0.024	$1.38 m_{pl}$	4.531×10^3
$P + W + hl + BAO$	0.9608 ± 0.0054	3.091 ± 0.025	$1.41 m_{pl}$	422.196

$$0.65 \times 10^{-8} \leq \nu_s \leq 2.45 \times 10^0 \text{ Hz, with } n \sim 2.1,$$

$$0.57 \times 10^3 \leq \nu_1 \leq 0.39 \times 10^6 \text{ Hz, with } n \sim 2.1, \quad (39)$$

for Planck. It is noted that the T_{rh} does not change the frequencies less than ν_s and ν_1 .

The obtained frequencies based on WMAP and Planck can affect on the shape of the spectrum of the waves in the range ν_s up to ν_1 and gives us interesting results about detection of the waves. We plot the spectrum in Figs.[1, 2] compared to the sensitivity of detectors such as LISA, BBO and ultimate-DECIGO (black, red and green colors respectively). The yellow, pink and blue colors are based on the initial amount of (ν_s, ν_1) , WMAP-9 and Planck data respectively. Moreover the dashed (solid) of pink and blue colors are based on the minimum (maximum) amount of (ν_s, ν_1) due to T_{rh} from eqs.(34, 35) in both figures. It is noted that there are some overlap in dashed yellow and solid blue for both figures.

It is shown that all detectors can detect the waves corresponds to the amount of T_{rh} in both figures. But, there is no chance for detection of the waves due to minimum amount of T_{rh} (dashed pink and blue colors) with all detectors while it exists for the maximum amount of T_{rh} for the ultimate-DECIGO (solid pink and blue colors) in Figs.[1, 2]. The same thing

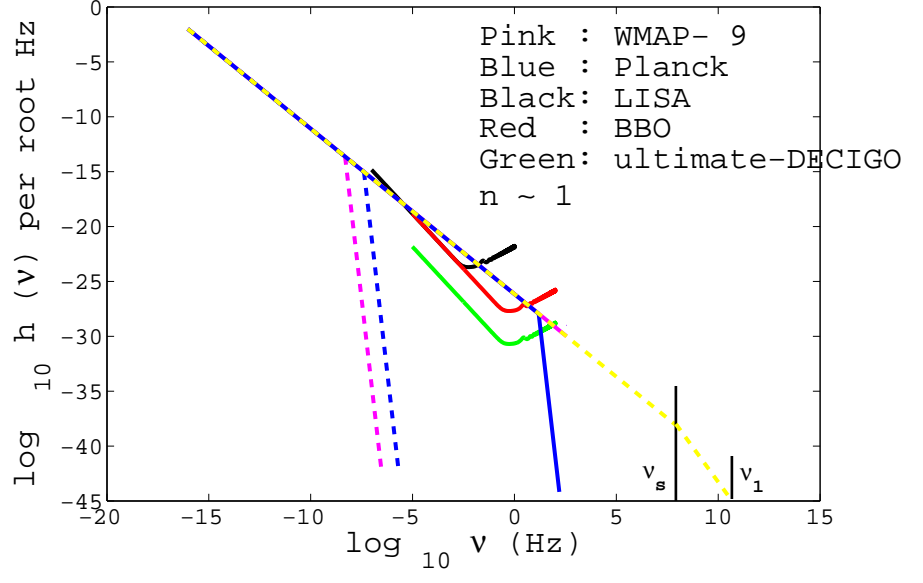


FIG. 1: The comparison of the spectrum of gravitational waves with the sensitivity of detectors such as LISA, BBO and ultimate-DECIGO for $n \sim 1$.

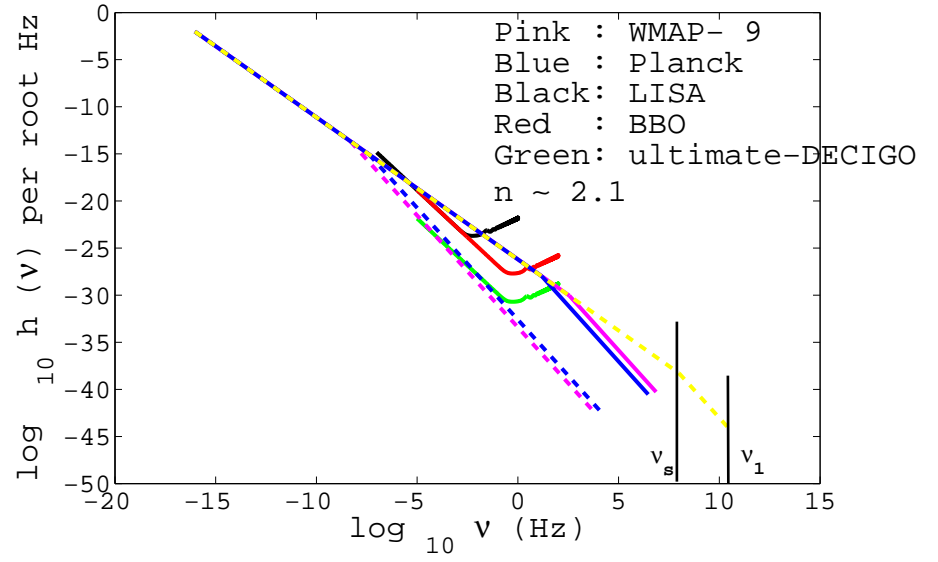


FIG. 2: The comparison of the spectrum of gravitational waves with the sensitivity of detectors such as LISA, BBO and ultimate-DECIGO for $n \sim 2.1$.

has happened about the detection of the waves due to minimum amount of T_{rh} for LISA and BBO in Fig.[2]. But there is some more chance about the detection of the waves in full range of the T_{rh} with ultimate DECIGO in Fig.[2]. It is noted that there are some other methods for the detection of the waves at very high frequency range ν_s up to ν_1 based on wave guide [28] and Gaussian beam [29].

Therefore based on this work, it is shown that the reheating era and reheating temperature play main role in the shape and evolution of the waves in the range ν_s up to ν_1 . Hence there is no reason for looking for the detection of gravitational waves in the high range (more than 10^3Hz and 10^6Hz correspond to the eqs.(36-39)). It is observed that there are some intersections between the spectrum and sensitivity of the detectors especially ultimate DECIGO in the mentioned range. Hence we hope to detect the waves with future mentioned mission of the detectors.

IV. DISCUSSION AND CONCLUSION

The relic gravitational waves that originated in the early universe are very important in cosmology because they carry information about the physical conditions of early evolution stages of the universe. The inflation era and subsequent stages of evolution of the universe played an important role on the spectrum of the waves. The reheating stage of the universe after inflation era is supposed to be an important evolution stage of the universe. The relic gravitational waves being used to determine the reheating temperature of the universe. Actually the reheating stage is model dependent but later a stage called z is included to consider a general reheating scenario in the context of relic gravitational waves. On the consideration of end of the inflationary stage and thermo nuclear synthesis, the reheating temperature is more than a few MeV and less than that 10^{16} GeV.

It is shown that the reheating stage and reheating temperature can be affect on the shape of the spectrum of the gravitational waves based on the WMAP-9 and Planck data. Therefore there is no reason for looking for detection of gravitational waves in the high range. The compared results between the sensitivity of LISA, BBO, ultimate-DECIGO and spectrum of the waves show that all detectors can detect the waves corresponds to the amount of T_{rh} . But, there is no chance for detection of the waves due to minimum amount of T_{rh} with LISA and BBO while it exists for the maximum amount of T_{rh} especially with

the ultimate-DECIGO in Fig.[1]. Also there is some more chance for detection of the waves in full range of the T_{rh} with ultimate-DECIGO in Fig.[2]. Hence the future mission of the mentioned detectors especially ultimate-DECIGO are likely to detect the waves.

Appendix A

The tensor to scalar ratio is $r = P_T(k)/P_S(k)$, where the tensor power spectrum $P_T(k)$ and scalar power spectrum $P_S(k)$ are as follows

$$P_T(k) = P_T(k_0^p) \left(\frac{k}{k_0^p}\right)^{n_t(k_0^p) + \frac{1}{2}\alpha_t \ln(k/k_0^p)}, \quad (\text{A1})$$

$$P_S(k) = P_S(k_0^p) \left(\frac{k}{k_0^p}\right)^{n_s(k_0^p) - 1 + \frac{1}{2}\alpha_s \ln(k/k_0^p)}. \quad (\text{A2})$$

The parameters n_t, n_s are tensor and scalar spectral indexes respectively with their corresponding running $\alpha_t \equiv dn_t/d \ln k$ and $\alpha_s \equiv dn_s/d \ln k$ and also $k_0^p = 0.002 \text{ Mpc}^{-1}$ is pivot wave number [30].

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